Energy Spectrum of Neutral Collective Excitations in Striped Hall States

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We investigate neutral collective excitations in the striped Hall state. In the striped Hall state, the magnetic translation and rotation symmetries are spontaneously broken. Using the commutation relation between charges and currents corresponding to the broken and unbroken symmetry, the existence of the gapless neutral excitation is proved. The spectrum of the neutral collective excitation at the half-filled third Landau level is obtained in the single mode approximation. We find the periodic line nodes in the spectrum. The spectrum is compared with the particle-hole excitation spectrum in the Hartree-Fock approximation.

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I. INTRODUCTION

The two-dimensional electron system under a strong magnetic field shows astonishing diversity in experiments and theories. The quantized Hall conductance[1,2] is observed in the incompressible quantum liquid state.[3] Fractional quantum Hall states (FQHS) have anyonic quasiparticles in the excited states.[4] Around the half-filled lowest Landau level, the composite Fermi liquid state[5] which has an isotropic Fermi surface is observed. At the half-filled second Landau level, on the other hand, the fractionally quantized Hall conductance is observed;[6] it is suggested that the state has an energy gap due to the Cooper pairing of the electrons.[7,8]

Recently, highly anisotropic states, which have extremely anisotropic longitudinal resistances, have been observed around the half-filled third and higher Landau levels.[9,10] It is believed that the anisotropic state is the striped Hall state which is a unidirectional charge density wave in the mean field theory.[11,12] The anisotropy of the resistance is naturally explained by the anisotropic Fermi surface in the magnetic Brillouin zone.[13,14] It was predicted that fluctuation effects turn the striped Hall state into the smectic or nematic liquid crystal.[15–20] A low energy theory was studied in the coupled Luttinger liquid picture using the Hartree-Fock approximation.[21] A peculiar response to external modulations was predicted in a perturbation theory[22] and numerical calculations.[23]

In the highly anisotropic states, underlying symmetries are spontaneously broken and Nambu-Goldstone (NG) modes should appear. It is important to find the property of the NG mode. In this paper, we investigate the property of the NG mode in striped Hall states. In the two-dimensional system under a uniform magnetic field, the system has the magnetic translation and rotation symmetry. In the striped Hall state, a magnetic translation in one direction is broken to the discrete translation and the rotation is also broken to the π-rotation, spontaneously. We prove the Goldstone theorem for the striped Hall state using the commutation relation between charges and currents corresponding to the broken and unbroken symmetry. The theorem shows that a gapless neutral excitation exists and couples with the density operator. The spectrum of the neutral collective excitation is obtained in the single mode approximation numerically at the half-filled third Landau level. The spectrum has a multiple line node structure and cusps. Furthermore, the spectrum has anisotropic feature, that is, in one direction it resembles the liquid Helium spectrum with the phonon and roton minimum, and in another direction it resembles the FQHS spectrum. To show the validity of the single mode approximation in the present system, we compare the spectrum with the particle-hole excitation spectrum in the Hartree-Fock approximation.

The paper is organized as follows. Conserved currents in the two-dimensional system under a uniform magnetic field are clarified in Sec. II. In Sec. III, a mean field theory for the striped Hall state is presented. Goldstone theorem for the striped Hall state is proved in Sec. IV. The spectrum for the neutral collective excitation in the striped Hall state is obtained in the single mode approximation and compared with the particle-hole excitation spectrum in the Hartree-Fock approximation in Sec. V. Summary is given in Sec. VI.

II. CONSERVED CURRENTS IN A UNIFORM MAGNETIC FIELD

Let us consider the two-dimensional electron system in a uniform magnetic field $B = \partial_x A_y - \partial_y A_x$. We ignore the spin degree of freedom and use the natural unit ($\hbar = c = 1$) in this paper. We introduce two sets of coordinates, relative coordinates and guiding center coordinates. The relative coordinates are defined by

$$\xi = \frac{1}{eB}(-i\partial_y + eA_y),\ \eta = -\frac{1}{eB}(-i\partial_x + eA_x). \quad (2.1)$$

The guiding center coordinates are defined by

$$X = x - \xi,\ Y = y - \eta. \quad (2.2)$$

These coordinates satisfy the following commutation relations,

$$[X,Y] = -[\xi,\eta] = i/eB,$$
$$[X,\xi] = [X,\eta] = [Y,\xi] = [Y,\eta] = 0. \tag{2.3}$$

The operators X and Y are the generators of the magnetic translations of the one-particle state in $-y$ direction and x direction respectively. The angular momentum J is written as

$$J = \frac{eB}{2}(\xi^2 + \eta^2 - X^2 - Y^2). \tag{2.4}$$

J is the generator of the rotation of the one-particle state.

The total Hamiltonian H for the interacting charged particles is the sum of the free Hamiltonian H_0 and the Coulomb interaction Hamiltonian $H_{\rm int}$ as follows,

$$\begin{aligned} H &= H_0 + H_{\rm int}, \\ H_0 &= \int d\mathbf{r}\Psi^\dagger(\mathbf{r})\frac{m\omega_c^2}{2}(\xi^2+\eta^2)\Psi^\dagger(\mathbf{r}), \\ H_{\rm int} &= \frac{1}{2}\int d\mathbf{r}d\mathbf{r}'\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\Psi(\mathbf{r}')\Psi(\mathbf{r}), \end{aligned} \tag{2.5}$$

where Ψ is the electron field operator, $\omega_c = eB/m$ and $V(\mathbf{r}) = q^2/r$ ($q^2 = e^2/4\pi\epsilon$, ϵ is the dielectric constant) for the Coulomb potential.

Conserved charges are obtained as the spatial integral of the zeroth component of the Noether currents for the symmetries. We define the conserved charges, Q, Q_X, Q_Y, and Q_J for U(1), magnetic translations, and rotation symmetry respectively, as follows,

$$\begin{aligned} Q &= \textstyle\int j^0(\mathbf{r})d\mathbf{r}, \\ Q_X &= \textstyle\int j_X^0(\mathbf{r})d\mathbf{r},\ Q_Y = \int j_Y^0(\mathbf{r})d\mathbf{r}, \\ Q_J &= \textstyle\int j_J^0(\mathbf{r})d\mathbf{r}. \end{aligned} \tag{2.6}$$

Noether currents are defined by

$$\begin{aligned} j^\mu &= {\rm Re}(\Psi^\dagger v^\mu \Psi), \\ j_X^\mu &= {\rm Re}(\Psi^\dagger v^\mu X\Psi) - \frac{1}{eB}\delta_y^\mu\mathcal{L}, \\ j_Y^\mu &= {\rm Re}(\Psi^\dagger v^\mu Y\Psi) + \frac{1}{eB}\delta_x^\mu\mathcal{L}, \\ j_J^\mu &= {\rm Re}(\Psi^\dagger v^\mu J\Psi) + \epsilon_{0\mu i}x^i\mathcal{L}, \end{aligned} \tag{2.7}$$

where $v^\mu = (1,\mathbf{v})$, $\mathbf{v} = \omega_c(-\eta,\xi)$, Re means real part, and $\mathcal{L}$ is the Lagrangian density for the total Hamiltonian H. These charges commute with the total Hamiltonian H and obey the following commutation relations,

$$\begin{aligned} [Q_X,Q_Y] &= \frac{i}{eB}Q, \\ [Q_J,Q_X] &= iQ_Y, \\ [Q_J,Q_Y] &= -iQ_X. \end{aligned} \tag{2.8}$$

The U(1) charge Q commutes with all conserved charges. We assume that Q is not broken and the ground state is the eigenstate of Q as $Q|0\rangle = N_e|0\rangle$ where N_e is a number of electrons. It should be noted that these symmetries are not broken in the Laughlin state.[3] Hence the NG mode does not exist. The Laughlin state is the eigenstate of Q_J and annihilated by $Q_X + iQ_Y$.

The commutation relations between the conserved charges and the current density operators read,

$$\begin{aligned} [Q_X,j^\mu] &= -\frac{i}{eB}\partial_y j^\mu,\ [Q_Y,j^\mu] = \frac{i}{eB}\partial_x j^\mu, \\ [Q_X,j_X^\mu] &= -\frac{i}{eB}\partial_y j_X^\mu,\ [Q_Y,j_X^\mu] = \frac{i}{eB}\partial_x j_X^\mu - \frac{i}{eB}j^\mu, \\ [Q_X,j_Y^\mu] &= -\frac{i}{eB}\partial_y j_Y^\mu + \frac{i}{eB}j^\mu,\ [Q_Y,j_Y^\mu] = \frac{i}{eB}\partial_x j_Y^\mu. \end{aligned} \tag{2.9}$$

The U(1) charge Q commutes with all Noether currents. If the expectation value of the right hand side of these equation is not zero, corresponding symmetry is spontaneously broken. In the striped Hall state discussed in the next section, $\partial_x\langle j^\mu\rangle$ is not zero and the magnetic translation symmetry in x direction is spontaneously broken.

III. SYMMETRY BREAKING IN THE STRIPED HALL STATE

In this section, we study the self-consistent solution for the striped Hall state in the Hartree-Fock approximation. The state breaks the translation and rotational symmetry spontaneously.

We use the von Neumann lattice (vNL) base for the one-particle states.[24] A discrete set of coherent states of guiding center coordinates,

$$\begin{aligned} (X+iY)|\alpha_{mn}\rangle &= z_{mn}|\alpha_{mn}\rangle, \\ z_{mn} &= a(mr_s + i\tfrac{n}{r_s}),\ m,\ n;\ \text{integers}, \end{aligned} \tag{3.1}$$

is a complete set of the (X,Y) space. These coherent states are localized at the position $a(mr_s, n/r_s)$, where a positive real number r_s is the asymmetric parameter of the unit cell. By Fourier transforming these states, we obtain the orthonormal basis in the momentum representation

$$\begin{aligned} |\beta_{\mathbf{p}}\rangle &= \textstyle\sum_{mn} e^{ip_x m + ip_y n}|\alpha_{mn}\rangle/\beta(\mathbf{p}), \\ \beta(\mathbf{p}) &= (2{\rm Im}\tau)^{1/4}e^{\frac{i\tau p_y^2}{4\pi}}\vartheta_1(\tfrac{p_x+\tau p_y}{2\pi}|\tau), \end{aligned} \tag{3.2}$$

where ϑ_1 is a Jacobi's theta function and $\tau = ir_s^2$. The two-dimensional lattice momentum $\mathbf{p}$ is defined in the Brillouin zone (BZ) $|p_i| < \pi$. Next we introduce a complete set in (ξ,η) space, that is the eigenstate of the one-particle free Hamiltonian,

$$\frac{m\omega_c^2}{2}(\xi^2+\eta^2)|f_l\rangle = \omega_c(l+\frac{1}{2})|f_l\rangle. \tag{3.3}$$

The Hilbert space is spanned by the direct product of these eigenstates

$$|l,\mathbf{p}\rangle = |f_l\rangle \otimes |\beta_{\mathbf{p}}\rangle, \tag{3.4}$$

where l is the Landau level index and $l = 0,\ 1,\ 2,\ \cdots$. We set $a = 1$ ($eB = 2\pi$) in the following calculation for simplicity.

Electron field operator is expanded by the vNL base as

$$\Psi(\mathbf{r}) = \sum_{l=0}^{\infty} \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l(\mathbf{p}) \langle \mathbf{r}|l, \mathbf{p}\rangle, \tag{3.5}$$

where b_l is the anti-commuting annihilation operator. $b_l(\mathbf{p})$ obeys the non-trivial boundary condition, $b_l(\mathbf{p} + 2\pi\mathbf{N}) = e^{i\phi(p,N)} b_l(\mathbf{p})$, where $\phi(p, N) = \pi(N_x + N_y) - N_y p_x$ and $\mathbf{N} = (N_x, N_y)$ are intergers. The Fourier transform of the density $\rho(\mathbf{k}) = \int d\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} j^0(\mathbf{r})$ is written as

$$\begin{aligned}\rho(\mathbf{k}) = \sum_{ll'} \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_{l'}(\mathbf{p} - \hat{\mathbf{k}}) \langle f_l | e^{ik_x\xi + ik_y\eta} | f_{l'}\rangle \\ \times e^{-\frac{i}{4\pi}\hat{k}_x(2p_y - \hat{k}_y)},\end{aligned} \tag{3.6}$$

where $\hat{\mathbf{k}} = (r_s k_x, k_y/r_s)$. Conserved charges are written in the momentum representation as

$$\begin{aligned} Q &= \sum_l \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}), \\ Q_X &= r_s \sum_l \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) (i\frac{\partial}{\partial p_x} - \frac{p_y}{2\pi}) b_l(\mathbf{p}), \\ Q_Y &= \frac{1}{r_s} \sum_l \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) i\frac{\partial}{\partial p_y} b_l(\mathbf{p}), \\ Q_J &= \sum_l \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) [l + \frac{1}{2} \\ &\quad + \pi\{r_s^2 (i\frac{\partial}{\partial p_x} - \frac{p_y}{2\pi})^2 + r_s^{-2} (i\frac{\partial}{\partial p_y})^2\}] b_l(\mathbf{p}). \end{aligned} \tag{3.7}$$

As seen in Eq. (3.7), the magnetic translation in the real space is equivalent to the magnetic translation in the momentum space. The free kinetic energy is quenched in a magnetic field and the one-particle spectrum becomes flat. Therefore the free system in a magnetic field is translationally invariant in the momentum space. We show that there exists the Fermi surface in the mean field solution for the striped Hall state. The existence of the Fermi surface indicates the violation of the translational symmetry in the momentum space or in the real space.

The mean field state is constructed as

$$|0\rangle = N_1 \prod_{\mathbf{p}\in \mathrm{F.S.}} b_l^\dagger(\mathbf{p}) |\mathrm{vac}\rangle, \tag{3.8}$$

where F.S. means Fermi sea, N_1 is a normalization constant, and $|\mathrm{vac}\rangle$ is the vacuum state in which the $l-1$ th and lower Landau levels are fully occupied. The mean field of the two-point function for the electron field is given by

$$\begin{aligned}\langle 0|b_l^\dagger(\mathbf{p}) b_{l'}(\mathbf{p}')|0\rangle = \delta_{ll'}\theta(\mu - \epsilon_{\mathrm{HF}}(\mathbf{p})) \\ \times \sum_N (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}' + 2\pi\mathbf{N}) e^{i\phi(p,N)},\end{aligned} \tag{3.9}$$

where μ is a chemical potential and one-particle energy $\epsilon_{\mathrm{HF}}(\mathbf{p})$ is determined self-consistently in the Hartree-Fock approximation.[13,14] We assume that the magnetic field B is so strong that Landau level mixing effects can be neglected.[25] Then we use the Hamiltonian projected to the l th Landau level

$$\begin{aligned} P_l H P_l &= \omega_c N_e^* (l + \frac{1}{2}) + H^{(l)}, \\ H^{(l)} &= P_l H_{\mathrm{int}} P_l, \end{aligned} \tag{3.10}$$

where P_l is the projection operator to the l th Landau level and N_e^* is a number of electrons occupying the l th Landau level. The free kinetic term is quenched and the total Hamiltonian is reduced to the Coulomb interaction term projected to the l th Landau level, $H^{(l)}$.

It was shown that the mean field state with Fermi sea $|p_y| < \pi\nu_*$ satisfies the self-consistency equation at the filling factor $\nu = l + \nu_*$ ($0 < \nu_* < 1$) in the Hartree-Fock approximation[13,14] and corresponds to the striped Hall state whose density is uniform in y direction and periodic with a period r_s in x direction.[14] The one-particle energy $\epsilon_{\mathrm{HF}}(\mathbf{p})$ depends only on p_y in this self-consistent solution. The Fermi sea and corresponding charge density distribution in x-y space are sketched in Fig. 1. The period of stripe is r_s. The electric current flows along each stripes. Note that the true charge density and current density distribution in the mean field theory are not sharp as shown in the figure but fuzzy.[14] As seen in Fig. 1 (b), $\partial_x\langle j^0\rangle$ and $\partial_x\langle j^y\rangle$ are not zero and the magnetic translation in x-direction and rotation symmetry are spontaneously broken, but a finite magnetic translation in x-direction by the period of stripe r_s and π rotation symmetry are preserved.

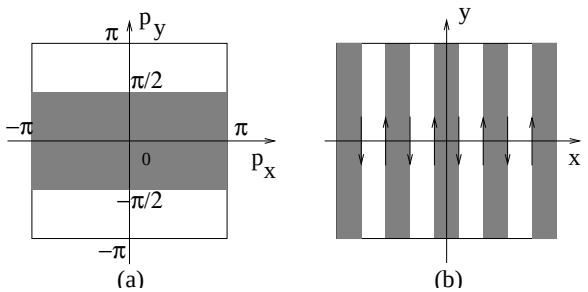

Fig. 1. (a) Fermi sea for the striped Hall state at the half-filling. (b) Schematical charge density distribution in x-y space. The arrows indicate the direction of electric current.

Hartree-Fock energy per area for the striped Hall state is calculated as

$$\begin{aligned} E_{\mathrm{HF}} &= \langle 0|H^{(l)}|0\rangle/L^2 \\ &= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{dp_y}{2\pi} \epsilon_{\mathrm{HF}}(p_y), \end{aligned} \tag{3.11}$$

where L^2 is an area of the system. $E_{\rm HF}$ depends on the period of the stripe r_s and the optimal value of r_s is determined by minimizing $E_{\rm HF}$. At the filling factor $\nu = 2 + \frac{1}{2}$, the optimal value is $r_s^* = 2.474$.[14]

IV. GOLDSTONE THEOREM FOR THE STRIPED HALL STATE

In this section, we assume that the magnetic translation in x direction and rotation symmetries are broken, and the magnetic translation in y direction, finite translation in x direction with a period r_s and π rotation are preserved in the striped Hall state. The Goldstone theorem for the striped Hall state is proven under these general assumptions.

For definite discussions, the periodic boundary condition in (X, Y) space is imposed for $\langle\alpha_{mn}|\beta_{\mathbf{p}}\rangle$ and $\mathbf{p}$ is discretized as $\mathbf{p} = (2\pi n_x/L_x, 2\pi n_y/L_y)$, where n_x, n_y are integers and L_x, L_y are numbers of lattice sites in the x direction and y direction, respectively. Therefore the well-defined translation operators in the Hilbert space are $e^{i2\pi n_x Q_X/r_s L_x}$ and $e^{i2\pi n_y r_s Q_Y/L_y}$. For the striped Hall state, unbroken operators are Q, $e^{i2\pi Q_X/r_s L_x}$, $e^{i2\pi r_s Q_Y}$ and $e^{i\pi Q_J}$. Then we can specify the state using the eigenvalues of the unbroken operators as

$$\begin{aligned} H|n\rangle &= E_n|n\rangle, \\ e^{i2\pi\frac{Q_X}{r_s L_x}}|n\rangle &= e^{i2\pi\frac{Q_X^{(n)}}{r_s L_x}}|n\rangle, \\ e^{i2\pi r_s Q_Y}|n\rangle &= e^{i2\pi r_s Q_Y^{(n)}}|n\rangle, \\ Q|n\rangle &= N_e|n\rangle, \end{aligned} \tag{4.1}$$

where $n \geq 0$ and $|0\rangle$ is the ground state for the striped Hall state. For the ground state $|0\rangle$, we can show that $e^{i2\pi Q_X^{(0)}/r_s L_x} = \pm 1$ and $e^{i2\pi r_s Q_Y^{(0)}} = \pm 1$. We can also show that $e^{i\pi Q_J}|0\rangle = \pm|0\rangle$ (See Appendix A). The thermodynamic limit L_x, $L_y \to \infty$ should be taken at the last stage of calculations. For example, Q_X is defined by $\lim_{L_x\to\infty}\frac{r_s L_x}{2\pi i}(e^{i2\pi Q_X/r_s L_x} - 1)$. Since Q_X and Q_Y correspond to the magnetic translation in $-y$ and x direction respectively, the eigenvalues $Q_X^{(n)}$ and $Q_Y^{(n)}$ are regarded as a kind of momentum in $-y$ and x direction respectively.

Let us show the Goldstone theorem for the striped Hall state. An expectation value for $|0\rangle$ of the commutation relations Eq. (2.9) for the broken charge Q_Y reads

$$\begin{aligned} \frac{i}{2\pi}\partial_x\langle j^0(\mathbf{r},t)\rangle &= \langle 0|[Q_Y, j^0(\mathbf{r},t)]|0\rangle \\ &= \int d\mathbf{r}'\langle 0|[j_Y^0(\mathbf{r}',t'), j^0(\mathbf{r},t)]|0\rangle \\ &= \sum_n \int d\mathbf{r}'\{\langle 0|j_Y^0(\mathbf{r}',t')|n\rangle\langle n|j^0(\mathbf{r},t)|0\rangle \\ &\qquad - h.c.\}, \end{aligned} \tag{4.2}$$

where $h.c.$ means hermitian conjugate. The left-hand side of this equation is not zero and does not depend on time t. The right-hand side of this equation is rewritten by using translation operators in t and $\mathbf{r}$. Note that j_Y^0 is transformed covariantly under the magnetic translation in y direction, that is,

$$\begin{aligned} e^{i2\pi\Delta y Q_X} j_Y^0(\mathbf{r}) e^{-i2\pi\Delta y Q_X} &= j_Y^0(\mathbf{r} + (0,\Delta y)) \\ &\quad -\Delta y j^0(\mathbf{r} + (0,\Delta y)). \end{aligned} \tag{4.3}$$

In the x direction, we use the notation $\mathbf{r} = (N_x r_s + \bar{x}, y)$, where N_x is an integer and $0 < \bar{x} < r_s$. Using the relation $[Q, j^0] = 0$ and Eqs. (4.1) and (4.3), the right-hand side of Eq. (4.2) is rewritten as

$$\begin{aligned} &\sum_{n>0}\int_0^{r_s}\frac{d\bar{x}'}{r_s}[\{\langle 0|j_Y^0((\bar{x}',0),0)|n\rangle\langle n|j^0((\bar{x},0),0)|0\rangle \\ &\times e^{i(t'-t)(E_n-E_0)} - h.c.\}\delta(Q_X^{(0)} - Q_X^{(n)})\delta_{\frac{1}{r_s}}(Q_Y^{(0)} - Q_Y^{(n)}) \\ &+\frac{1}{2\pi i}\{\langle 0|j^0((\bar{x}',0),0)|n\rangle\langle n|j^0((\bar{x},0),0)|0\rangle \\ &\times e^{i(t'-t)(E_n-E_0)} + h.c.\}\delta'(Q_X^{(0)} - Q_X^{(n)})\delta_{\frac{1}{r_s}}(Q_Y^{(0)} - Q_Y^{(n)})] \end{aligned} \tag{4.4}$$

where $\delta_{\frac{1}{r_s}}(Q_Y^{(0)} - Q_Y^{(n)})$ is a periodic Dirac's delta-function with a period $1/r_s$. The derivative of the Dirac's delta-function appears in the second term. We assume the completeness $1 = \sum_n |n\rangle\langle n| = |0\rangle\langle 0| + \sum_\alpha \int d\mathbf{q}|\mathbf{q},\alpha\rangle\langle\mathbf{q},\alpha|$. Here $|\mathbf{q},\alpha\rangle$ is an excited state with eigenvalues $\mathbf{Q}^{(n)} = \mathbf{Q}^{(0)} + \mathbf{q}$, $E_n = E_0 + \Delta E_\alpha(\mathbf{q})$, and species index α. Then Eq. (4.4) is rewritten as

$$\begin{aligned} &\sum_\alpha\int_0^{r_s}\frac{d\bar{x}'}{r_s}[\{\langle 0|j_Y^0((\bar{x}',0),0)|\mathbf{0},\alpha\rangle\langle\mathbf{0},\alpha|j^0((\bar{x},0),0)|0\rangle \\ &\times e^{i(t'-t)\Delta E_\alpha(\mathbf{0})} - h.c.\} \\ &+\{\frac{i}{2\pi}\frac{\partial}{\partial q_x}\langle 0|j^0((\bar{x}',0),0)|\mathbf{q},\alpha\rangle\langle\mathbf{0},\alpha|j^0((\bar{x},0),0)|0\rangle \\ &\times e^{i(t'-t)\Delta E_\alpha(\mathbf{0})} - h.c.\}_{\mathbf{q}=0}], \end{aligned} \tag{4.5}$$

where we use the relation $\langle 0|Q|\mathbf{q},\alpha\rangle = 0$. The left-hand side of Eq. (4.2) does not depend on t and equals Eq. (4.5). Hence there exists the NG mode whose excitation energy $\Delta E_{\rm NG}(\mathbf{q})$ goes to zero as $\mathbf{q} \to 0$. Since the matrix elements in Eq. (4.5) must not be zero, NG mode $|\mathbf{q}, {\rm NG}\rangle$ belongs to the neutral charged sector and couples with the striped Hall state through the density operator, that is,

$$\langle\mathbf{q}, {\rm NG}|j^0|0\rangle \neq 0, \tag{4.6}$$

in $\mathbf{q} \to 0$.

This NG mode is a phonon due to breaking of a magnetic translation. NG modes (phonon)[26] also appear in the Wigner crystal in the absence of a magnetic field. In this case, the second term in Eq. (4.5) is absent. It would be interesting if this term is observed in the quantum Hall system. In the next section, we investigate the excitation spectrum using the density operator j^0 which couples with the NG mode.

V. NEUTRAL COLLECTIVE EXCITATIONS IN THE STRIPED HALL STATE

In this section we calculate the spectrum for a neutral collective excitation at the half-filled third Landau level using the single mode approximation. This method was used for the Liquid Helium by Feynman first[27] and applied to the FQHS later.[28] The single mode approximation is successful in the FQHS because the backflow problem is absent for the electron states projected to the l th Landau level.

First we define the Fourier transformed density operator in guiding center coordinates as

$$\rho_*(\mathbf{k}) = P_l \int d\mathbf{r}\Psi^\dagger(\mathbf{r})e^{ik_x X+ik_y Y}\Psi(\mathbf{r})P_l \tag{5.1}$$
$$= \int_{\rm BZ}\frac{d^2p}{(2\pi)^2}b_l^\dagger(\mathbf{p})b_l(\mathbf{p}-\hat{\mathbf{k}})e^{-\frac{i}{4\pi}\hat{k}_x(2p_y-\hat{k}_y)}.$$

Density operator $\rho(\mathbf{k})$ is related to $\rho_*(\mathbf{k})$ as $P_l\rho(\mathbf{k})P_l = e^{-k^2/8\pi}L_l(k^2/4\pi)\rho_*(\mathbf{k})$, where L_l is the Laguerre polynomial. Using $\rho_*(\mathbf{k})$, $H^{(l)}$ is written as

$$H^{(l)} = \frac{1}{2}\int\frac{d^2k}{(2\pi)^2}\rho_*(\mathbf{k})v_l(k)\rho_*(-\mathbf{k}), \tag{5.2}$$

where $v_l(k) = e^{-k^2/4\pi}(L_l(k^2/4\pi))^2 2\pi q^2/k$ for the Coulomb potential. It is well-known that the density operators projected to the Landau level are non-commutative, that is,

$$[\rho_*(\mathbf{k}),\rho_*(\mathbf{k}')] = -2i\sin\left(\frac{\mathbf{k}\times\mathbf{k}'}{4\pi}\right)\rho_*(\mathbf{k}+\mathbf{k}'). \tag{5.3}$$

Using this relation, we obtain the following commutation relations,

$$[Q_X,\rho_*(\mathbf{k})] = -\frac{k_y}{2\pi}\rho_*(\mathbf{k}), \tag{5.4}$$
$$[Q_Y,\rho_*(\mathbf{k})] = \frac{k_x}{2\pi}\rho_*(\mathbf{k}).$$

Therefore the state defined by

$$|\mathbf{k}\rangle = \rho_*(\mathbf{k})|0\rangle \tag{5.5}$$

is an eigenstate of $e^{i2\pi Q_X/r_sL_x}$ and $e^{i2\pi r_sQ_Y}$ with eigenvalues $e^{i2\pi(Q_X^{(0)}-\frac{k_y}{2\pi})/r_sL_x}$ and $e^{i2\pi r_s(Q_Y^{(0)}+\frac{k_x}{2\pi})}$ respectively. The quantum number $\mathbf{q}$ for the excited state is related to $\mathbf{k}$ as $k_i = 2\pi\epsilon^{ij}q_j$. We use the state $|\mathbf{k}\rangle$ as a neutral collective excitation state in the single mode approximation.

The variational excitation energy $\Delta(\mathbf{k})$ is written as

$$\Delta(\mathbf{k}) = \frac{\langle\mathbf{k}|(H^{(l)}-E_0)|\mathbf{k}\rangle}{\langle\mathbf{k}|\mathbf{k}\rangle} = \frac{f(\mathbf{k})}{s(\mathbf{k})},$$
$$f(\mathbf{k}) = \langle 0|[\rho_*(-\mathbf{k}),[H^{(l)},\rho_*(\mathbf{k})]]|0\rangle/2N_e^*, \tag{5.6}$$
$$s(\mathbf{k}) = \langle 0|\rho_*(-\mathbf{k})\rho_*(\mathbf{k})|0\rangle/N_e^*.$$

$s(\mathbf{k})$ is the so-called static structure factor. To derive these expressions, we use the relation $f(-\mathbf{k}) = f(\mathbf{k})$ and $s(-\mathbf{k}) = s(\mathbf{k})$ due to π rotation symmetry. See appendix A. Using the commutation relation (5.3), $f(\mathbf{k})$ is written as

$$f(\mathbf{k}) = 2\int\frac{d^2k'}{(2\pi)^2}v_l(k')\sin^2\left(\frac{\mathbf{k}'\times\mathbf{k}}{4\pi}\right) \tag{5.7}$$
$$\times\{s(\mathbf{k}+\mathbf{k}')-s(\mathbf{k}')\}$$

From Eq. (5.7) the variational excitation energy is calculable if we know the static structure factor $s(\mathbf{k})$. Using the mean field state (3.8) for the striped Hall state, $s(\mathbf{k})$ becomes

$$s(\mathbf{k}) = \frac{1}{\nu_*}\int_{\rm BZ}\frac{d^2p}{(2\pi)^2}\theta(\mu-\epsilon_{\rm HF}(p_y)) \tag{5.8}$$
$$\times\{1-\theta(\mu-\epsilon_{\rm HF}(p_y-\hat{k}_y))\}$$
$$+\sum_{N_x}\frac{(2\pi)^2}{\nu_*}\delta(\hat{k}_x+2\pi N_x)\delta(\hat{k}_y)\left(\frac{\sin(\pi\nu_*N_x)}{\pi N_x}\right)^2.$$

The first term behaves as $|k_y|/2\pi r_s\nu_*$ at small k_y and periodic in k_y direction with a period $2\pi r_s$. The second term has sharp peaks at $\mathbf{k} = (2\pi N_x/r_s, 0)$. These peaks correspond to the period for the stripe of the charge density. This behavior in the static structure factor was also obtained in the numerical calculation in a small system.[29,30] The numerical results for the energy spectrum Δ for $\nu = 2+1/2$ are shown in Figs. 2-4.

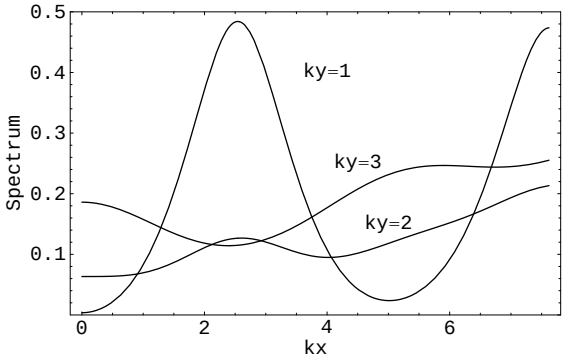

Fig. 2. Energy spectrum Δ at $0 < k_x < 6\pi/r_s$ for $k_y = 1, 2, 3$, $\nu = 2+1/2$ in the single mode approximation. Note that the spectrum remains finite at $k_x = 0$ for $k_y = 1$. The unit of $\mathbf{k}$ is a^{-1} and the unit of spectrum is q^2/a. The same units are used in Figs. 3-5.

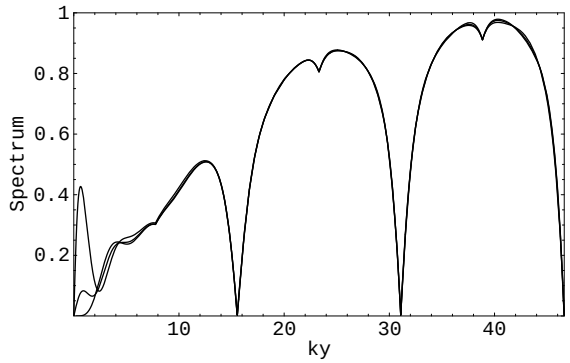

Fig. 3. Energy spectrum Δ at $0 < k_y < 6\pi r_s$ for $k_x = 0$, 1, 2, $\nu = 2 + 1/2$ in the single mode approximation. The energy gap vanishes at $k_y = 2\pi N_y r_s$ and cusps appear at $k_y = \pi(2N_y + 1)r_s$.

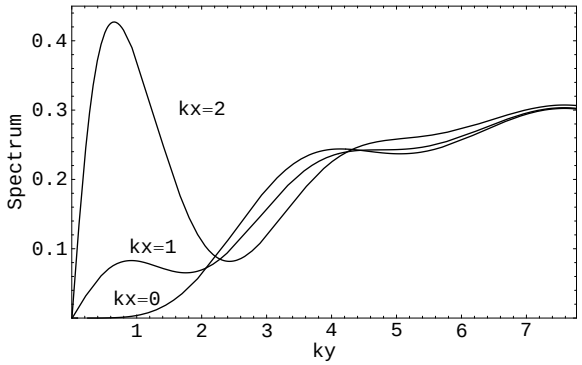

Fig. 4. Enlarged picture of the energy spectrum Δ at $0 < k_y < \pi r_s$ for $k_x = 0$, 1, 2, $\nu = 2 + 1/2$ in the single mode approximation.

As predicted by the Goldstone theorem in the previous section, the energy spectrum obtained by the single mode approximation is gapless at $\mathbf{k} = 0$. In fact, we can show that $\Delta(\mathbf{k})$ behaves as

$$\Delta(\mathbf{k}) = |k_y|(Ak_x^2 + Bk_y^4 + \mathrm{O}(k_x^2 k_y^2, k_x^4)), \tag{5.9}$$

at small $\mathbf{k}$ for a short-range potential. The reason for the absence of $|k_y|^3$ term is that the coefficient of $|k_y|^3$ vanishes by the condition $\frac{\partial E_{\mathrm{HF}}}{\partial r_s} = 0$ at $r_s = r_s^*$. For the Coulomb potential case, the logarithmic corrections are added in Eq. (5.9). See appendix B.

In addition to the gapless structure of the energy spectrum at $\mathbf{k} = 0$ due to NG mode, the results obtained by the single mode approximation present more rich structure. First we notice a periodic line node structure, that is, the energy spectrum vanishes at $k_y = 2\pi N_y r_s$. Except for $k_x = 0$, the dispersion at line node is linear. Second we notice that the energy spectrum has cusps at $k_y = \pi(2N_y + 1)r_s$. This is caused by the presence of the Fermi surface in the mean field state (3.8), which produces cusps in the first term of Eq. (5.8). Third the k_x dependence of the spectrum becomes weak at the large wave number. Fourth the spectrum has distinct features in different directions. Fig. 2 shows that the spectrum in k_x direction has a similarity with the collective excitation in FQHS[28] which has an energy gap at any wave number and magneto-roton structure. Fig. 4 shows that the spectrum in k_y direction has a similarity with the collective excitation in Liquid Helium which has phonon and roton spectrum. Fourth feature comes from the anisotropy of the Fermi sea. As seen in Fig. 1 (a), in p_x direction, the electron state is fully filled and has an energy gap. Therefore the system resembles FQHS in this direction. In p_y direction, on the other hand, there is a Fermi surface and electron state is gapless. First three features are understandable from comparison with the particle-hole excitation spectrum in the Hartree-Fock approximation.

For the large wave number $|k| > 2\pi/a$, it is expected that the excitation state becomes close to the free particle state. In the FQHS, the neutral excitation spectrum becomes close to the pair creation energy for the quasiparticle and quasi-hole.[31] In the striped Hall state, the one-particle energy is $\epsilon_{\mathrm{HF}}(p_y)$ in the Hartree-Fock approximation and the free part of Hamiltonian becomes

$$H_{\mathrm{HF}} = \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} \epsilon_{\mathrm{HF}}(p_y) b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}). \tag{5.10}$$

Therefore the excitation spectrum $\Delta_{\mathrm{ph}}(\mathbf{k})$ for the state of particle-hole pair $b_l^\dagger(\mathbf{p} + \hat{\mathbf{k}}) b_l(\mathbf{p})|0\rangle$ is bounded as

$$\Delta_{\mathrm{ph}}(\mathbf{k}) \geq |\epsilon_{\mathrm{HF}}(\pi\nu_* + \hat{k}_y) - \epsilon_{\mathrm{HF}}(\pi\nu_*)|. \tag{5.11}$$

$\pi\nu_*$ is a Fermi wave number. Lower bound of the particle-hole excitation spectrum for $\nu = 2 + 1/2$ is shown in Fig. 5.

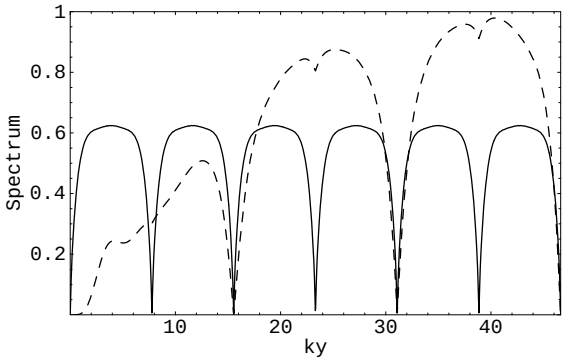

Fig. 5. Lower bound of the particle-hole excitation spectrum Δ_{ph} for $0 < k_y < 6\pi r_s$ (solid line). The spectrum vanishes at $k_y = \pi N_y r_s$. For comparison, the energy spectrum Δ for $k_x = 0$, $\nu = 2 + 1/2$ in the single mode approximation is also shown (dashed line).

As seen in this figure the particle-hole excitation spectrum vanishes and its slope diverges[13] at $k_y = \pi N_y r_s$. See Appendix B. This singularity is caused by the divergence of Fermi velocity $\partial\epsilon_{\mathrm{HF}}/\partial p_y$ at Fermi momentum. The spectrum Δ is smaller than Δ_{ph} and close to the Δ_{ph} near $k_y = 2\pi N_y r_s$ for a large wave number. The

existence of the Fermi surface leads to zero of $\Delta_{\rm ph}$ at $k_y = \pi(2N_y+1)r_s$, because the excitation energy for a hole inside the Fermi sea near the one Fermi surface and a particle outside the Fermi sea near the other Fermi surface is close to zero. The spectrum Δ is larger than $\Delta_{\rm ph}$ near $k_y = \pi(2N_y+1)r_s$ and has cusps. Hence the low energy excitation is well-approximated by the Δ near $k_y = 2\pi N_y r_s$ and by $\Delta_{\rm ph}$ near $k_y = \pi(2N_y+1)r_s$. In a usual Fermi system, the single mode approximation seems not to work well because of the particle-hole excitation near the Fermi surface at low energies. In the present case, however, the low energy excitation near the Fermi surface is suppressed by the divergence of Fermi velocity at Fermi surface. Hence the single mode approximation is expected to work well in the present system.

At a finite k_x, the spectrum is linear in k_y at small $\mathbf{k}$, which is similar to the one obtained by the hydrodynamics of quantum Hall smectics.[18] At $k_x = 0$, however, the spectrum is proportional to k_y^5, which disagrees with the results in the hydrodynamics of quantum Hall smectics. This discrepancy may be resolved by higher order correction to the static structure factor or by modifying the effective Hamiltonian in the hydrodynamics. At large $\mathbf{k}$, in which the hydrodynamic theory cannot be applied, the periodic line node structure is found in the single mode approximation for the first time.

VI. SUMMARY

We have studied neutral collective excitations in the striped Hall state. The Goldstone theorem has been proved in a system with the magnetic translation symmetry. We have obtained the neutral excitation spectrum in the striped Hall state using the single mode approximation. The spectrum has a new rich structure. The neutral collective mode includes the NG mode due to spontaneous breaking of magnetic translation symmetry. The spectrum is highly anisotropic, that is, the dispersion in k_x direction is similar to that of FQHS and the dispersion in k_y is similar to that of Liquid Helium. The spectrum is compared with the particle-hole excitation spectrum. The former has line nodes with a linear dispersion and the latter has line nodes with a diverging Fermi velocity. We hope these excitations will be observed in experiments for the evidence of the striped Hall state.

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APPENDIX A:

In this appendix, we prove the relation $e^{i\pi Q_J}|0\rangle = \pm|0\rangle$, $f(-\mathbf{k}) = f(\mathbf{k})$ and $s(-\mathbf{k}) = s(\mathbf{k})$ for the striped Hall state. As seen in Eq. (3.7), the generator for rotation is a free kinetic energy in a uniform magnetic field in the momentum space. Therefore it is convenient to expand the operator $b_l(\mathbf{p})$ by the eigenstate of the free kinetic energy as

$$b_l(\mathbf{p}) = \sum_{l'=0}^{\infty} b_{ll'}\psi_{l'}(\mathbf{p}), \qquad \text{(A1)}$$
$$\mathbf{D}^2\psi_{l'}(\mathbf{p}) = \frac{1}{\pi}(l'+\frac{1}{2})\psi_{l'}(\mathbf{p}),$$

where $\mathbf{D} = (r_s(i\frac{\partial}{\partial p_x} - \frac{p_y}{2\pi}), r_s^{-1} i\frac{\partial}{\partial p_y})$, $\psi_{l'}(\mathbf{p})$ is given by

$$\psi_{l'}(\mathbf{p}) = N_{l'}\sum_n H_{l'}(\frac{r_s(p_y+2\pi(n+\frac{1}{2}))}{\sqrt{2\pi}}) \qquad \text{(A2)}$$
$$\times e^{-\frac{r_s^2}{4\pi}(p_y+2\pi(n+\frac{1}{2}))^2} e^{i(n+\frac{1}{2})p_x + i\pi n},$$

$N_{l'}$ is a normalization constant, H_l is the Hermite polynomial, and $\psi_{l'}$ has a symmetry $\psi_{l'}(-\mathbf{p}) = (-1)^{l'+1}\psi_{l'}(\mathbf{p})$. Using this new base, Q_J reads

$$Q_J = \sum_{ll'} b_{ll'}^\dagger (l+l'+1) b_{ll'}. \qquad \text{(A3)}$$

Then we obtain the following relation,

$$e^{i\pi Q_J} b_l(\mathbf{p}) e^{-i\pi Q_J} = \sum_{l'} b_{ll'} e^{i\pi(l+l'+1)}\psi_{l'}(\mathbf{p}) \qquad \text{(A4)}$$
$$= (-1)^l b_l(-\mathbf{p}).$$

Since the Fermi sea for the striped Hall state is invariant under the π rotation, the ground state $|0\rangle$ is an eigenstate of $e^{i\pi Q_J}$ with an eigenvalue 1 or -1. Using Eq. (A4), one can prove that

$$e^{i\pi Q_J}\rho_*(\mathbf{k})e^{-i\pi Q_J} = \rho_*(-\mathbf{k}), \qquad \text{(A5)}$$
$$e^{i\pi Q_J} H^{(l)} e^{-i\pi Q_J} = H^{(l)}.$$

Inserting $e^{i\pi Q_J}$ and $e^{-i\pi Q_J}$ in the definition of $f(\mathbf{k})$ and $s(\mathbf{k})$, one can prove that $f(-\mathbf{k}) = f(\mathbf{k})$ and $s(-\mathbf{k}) = s(\mathbf{k})$ using Eq. (A5).

APPENDIX B:

In this appendix, we show that the anisotropic behavior of $\Delta(\mathbf{k})$ in Eq. (5.9) comes from the condition $\frac{\partial E_{\rm HF}}{\partial r_s}=0$.

Using the Fourier series expansion, the one-particle energy $\epsilon_{\rm HF}(p_y)$ and the Hartree-Fock energy are written as

$$\epsilon_{\rm HF}(p_y)=\sum_n v_{\rm HF}(\frac{2\pi n}{r_s})\frac{e^{inp_y}\sin\frac{\pi}{2}n}{\pi n}, \tag{B1}$$

$$E_{\rm HF}=\frac{1}{2}\sum_n v_{\rm HF}(\frac{2\pi n}{r_s})\left(\frac{\sin\frac{\pi}{2}n}{\pi n}\right)^2, \tag{B2}$$

$$v_{\rm HF}(k)=v_l(k)-\int d^2r v_l(2\pi r)e^{i\mathbf{k}\times\mathbf{r}}.$$

Furthermore, using Eq. (5.8), $f(\mathbf{k})$ in Eq. (5.7) is written as

$$f(\mathbf{k})=2\sum_{n=\rm odd}\{v_{\rm HF}(\sqrt{(\frac{2\pi n}{r_s}+k_x)^2+k_y^2})-v_{\rm HF}(\frac{2\pi n}{r_s})\}\times\left(\frac{\sin\frac{nk_y}{2r_s}}{\pi n}\right)^2. \tag{B3}$$

At $k_x=0$ and small k_y, it is easy to see that the coefficient of k_y^4 is proportional to $\frac{\partial E_{\rm HF}}{\partial r_s}$. Therefore $\frac{\partial E_{\rm HF}}{\partial r_s}=0$ leads to vanishing of the coefficient of k_y^4 at $k_x=0$ and the lowest order is k_y^6 at $k_x=0$. Since $s(\mathbf{k})$ is proportional to $|k_y|$ at small k_y, $\Delta(\mathbf{k})=\frac{f(\mathbf{k})}{s(\mathbf{k})}$ behaves as $|k_y|^5$ at $k_x=0$ and small k_y.

Note that $v_{\rm HF}(\mathbf{k})$ behaves as $-1/k$ at large k for the Coulomb potential. Hence the slope of $\epsilon_{\rm HF}(p_y)$ at $p_y=\pm\pi/2$ diverges logarithmically in Eq. (B1). In Eq. (B3), the Fourier transformation of $\mathrm{O}(k_x^2,k_y^2)n^{-5}$ with respect to k_y results in a term of $\mathrm{O}(k_x^2,k_y^2)k_y^4\log|k_y|$. Therefore the correction of $\mathrm{O}(k_x^2,k_y^2)|k_y|^3\log|k_y|$ is added in Eq. (5.9) for the Coulomb potential.

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